Economy and Application of Chaos Theory

1. Introduction

The theory of chaos came into being in solution of technical problems, where it describes the behaviour of nonlinear systems that have some hidden order, but still behave like systems controlled by chance. A linear model can describe a real system only if it is linear. If this is not fulfilled, then such models can simulate the real system only under ideal conditions, only for a short time. If a system is a nonlinear dynamic, a deterministic system it can generate not only the permanent trends and cycles, but it can also include random-looking behaviour. Processes of such behaviour are present in the economy. Chaos in the financial market is discussed in many books (Trippy, 1995), (Dostal, 2013). The behaviour of economic phenomena is a complex system (Rukovanský, 2005).

In this respect we can talk about two categories that are in opposition to each other: order and randomness. Chaos is something in between. It involves some degree of order. Some phenomena can appear random, but after the study of these phenomena some inner order can be discovered that governs these phenomena. For example, the movement of people at a railway station can appear accidental, but in fact it is behaviour with order controlled by the arrival and departure of trains. Also, the economy can exist in states of various degrees of order, chaos, and randomness. The behaviour of an economy is influenced quite often by natural disasters, political changes, etc.

In connection with chaos it is possible to speak about a so-called attractor, which represents an equilibrium position. An attractor is a condition towards which a dynamical system evolves over time. That is, points that get close enough to the attractor remain close even if slightly disturbed. Geometrically, an attractor can be a point, a curve, or even a complicated set with a fractal structure known as a chaotic attractor.

The geometrical explanation of an attractor could be done with the help of a pendulum. The attractor can be:

- Point - the stability is represented geometrically by a point. For example, when a pendulum is displaced, its final equilibrium position is reached when movement ceases.
- Cycle - the balance is represented geometrically by a limited cycle. For example, when a pendulum is moving with constant energy (potential energy plus kinetic energy = const.), its equilibrium position is reached when movement is cyclical.
- Chaotic - it is represented geometrically by order underlying the apparent chaos. For example, when the pendulum is driven by random energy, then the equilibrium position will be represented by a movement in zone (no point or cycle).

As we have said, the chaotic attractor can be presented geometrically in closed space, for example by a zone inside of two ellipses in two-dimensional space. See Figure 1.

The area of equilibrium position of a dynamic system could be explained by a zone between the outer and inner ellipses. The zone could represent for example the road for car racing. The trajectories of the cars are not the same, but very similar, because of reactions of drivers on outer conditions. Even if small accidents happen the cars stay on the road. If there will be a great accident, the cars leaves the road. The same is it with equilibrium of economy of states.

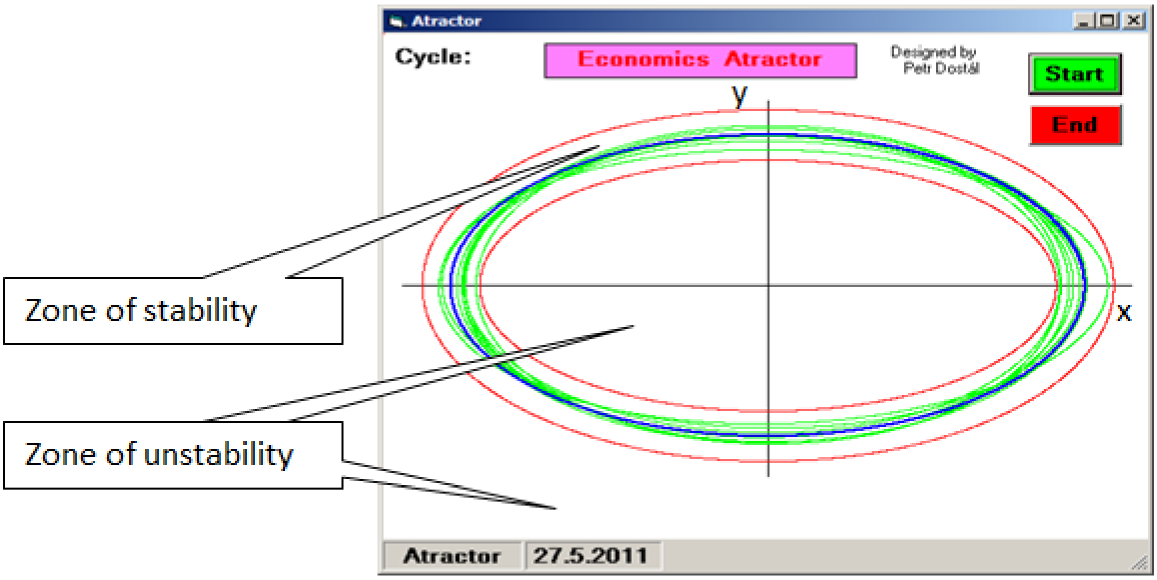

Figure 1: The presentation of chaotic attractor

2. Logistic function

The complicated behaviour in economy could be illustrated by the equation (in literature called a logistic function)

$x_{i+1} = r \,.\, x_i \,(1 - x_i),$

where r is a constant and x_i is a variable. The equation enables generation of behaviours that are considered to be point, periodic, and chaotic. Figure 2 presents the graph generated by this equation with initial values x_0 = 0.85 and r = 3.5 or r = 3.7. The vertical axis represents the values x_{i+1} and horizontal axis represents the values i. The curve in the graph represents chaotic behaviour, in that it has a hidden order that is described by the above-mentioned logistic equation, even if this reality is hardly recognizable.

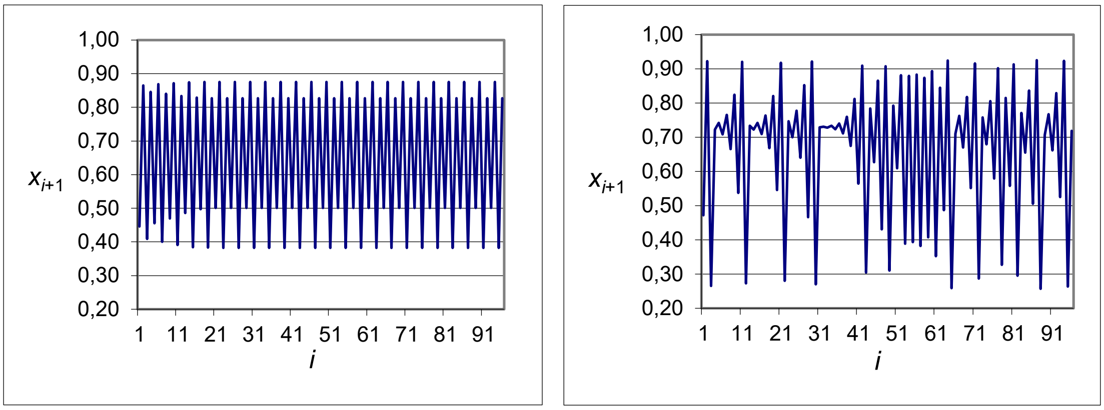

Figure 2: The graph of logistic function with parameters x_0 = 0.85 and r = 3.5 or 3.7

Application of logistic function in economy

The complicated behaviour in economy could be illustrated by the modified logistic function in the form

$$P_{t+1} = \frac{P_t}{1 + a\sigma_t} + (V_t - P_t),$$

where P_{t+1} simulates the market price of the share, σ_t is the volatility of yield of share, a is a coefficient of market aversion against risk, V_t is fundamental "inner" value of share in time t, $V_t - P_t$ is the response of market on the change of inner value.

The initial values (P_0, σ_0, V_0) could be different in the same way as the coefficients of market aversion against risk. If the value $\sigma = 0$ and market is neutral to risk $a = 0$, then $P_{t+1} = V_t$. Figure 3 presents graphically the course of P_{t+1} dependence on time with aversion to risk $a = 0.25$. The course of the curve reminds one of real values of time series of shares, commodities, currency ratios, and values of indices on the stock market.

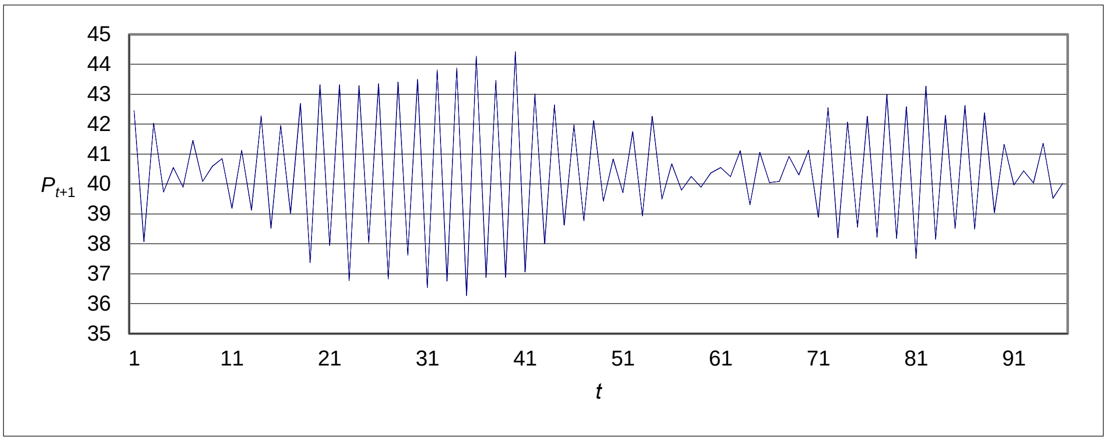

Figure 3: The graph of function P_{t+1} with parameter a = 0.25

3. Hurst exponent

The calculation of a Hurst exponent for a time series is done by the algorithm

$$\bar{x}(n) = \frac{1}{n}\sum_{i=1}^{n} x_i ,$$

$$X(t,n) = \sum_{i=1}^{t} (x_i - \bar{x}(\tau)), \qquad t = 1,2......, \tau;$$

$$S(n) = \sqrt{\frac{1}{n}\sum_{i=1}^{n} (x_i - \bar{x}(n))^2} ,$$

$$R(n) = \max_{1 \le t \le n} X(t,n) - \min_{1 \le t \le n} X(t,n) ,$$

$$H(n) = \log(R(n)/S(n))/\log(n) ,$$

where $x = (x_1, x_2, \ldots, x_n)$ is the sequence of values of time series and n is the number of values. The Hurst exponent is used for evaluation of order or randomness of dynamic systems that generate such time series. It could be for example for time series of records of GDP, unemployment, etc. The Hurst exponent reaches values in the range from 0 to 1. The value H around 0.5 means, that there is a long-term cycle in the time series. If the value *H* gets near to 0 or 1, it means that there is a long-term cycle in the time series and deterministic behavior is a part of time series.

Hurst exponent (H) determines the rate of chaos. If the probability distribution of the system is not normal, or closes enough, a nonparametric analysis is needed. Hurst exponent is such a method in the case of time series. Hurst exponent can distinguish fractal from random time series, or find the long memory cycles. If the value of H = 0.5, the time series is normally distributed, or has no long memory process. If ($H_i < H_{i+1}$), the time series is an antipersistent or mean reverting time series. If the time series has been up in the previous period, it is more likely to be going down in the next period, and vice versa. When ($H_i > H_{i+1}$), the time series is persistent or trend reinforcing time series. If the time series has been up in the previous period, it is more likely to be going up in the next period, and vice versa. Hurst exponent can measure how "jagged" a time series is. The lower the H value is, the more jagged the time series is. The higher the H value is, the more apparent the trend is, and the less jagged the time series is. The fractal dimension is calculated as a value = 2.0 – H. The constant c influences the value of H and it was set to 1.0.

Application of Hurst exponent in economy

There are shown some representative results of calculation of Hurst exponent concerning the of Microsoft, Co. share, title Figure 4 and NASDAQ index Figure 6 of the America stock market. The presented chart for $t = t_{max}$ and graphs confirm theory mentioned above.

The time series of Hurst exponent of MSFT share and NASDAQ index proves; that the more apparent the trend is, and the less jagged the time series is.

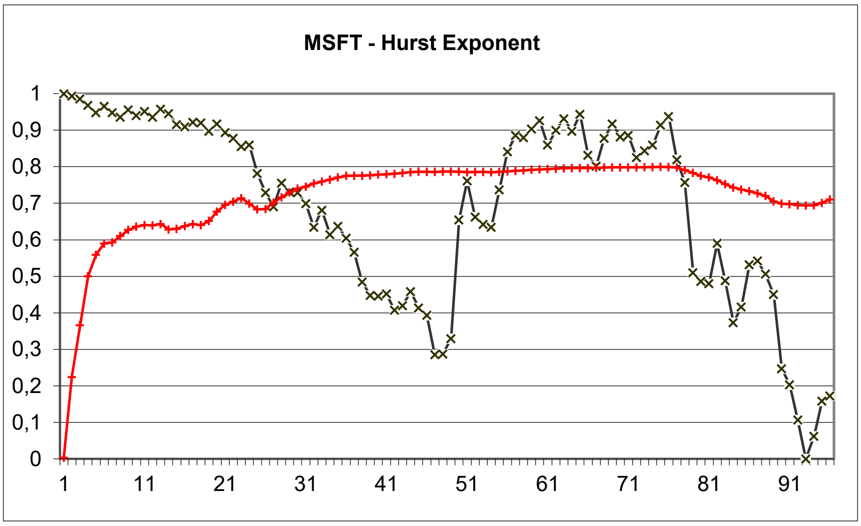

Figure 4: The graph MSFT - Hurst Exponent

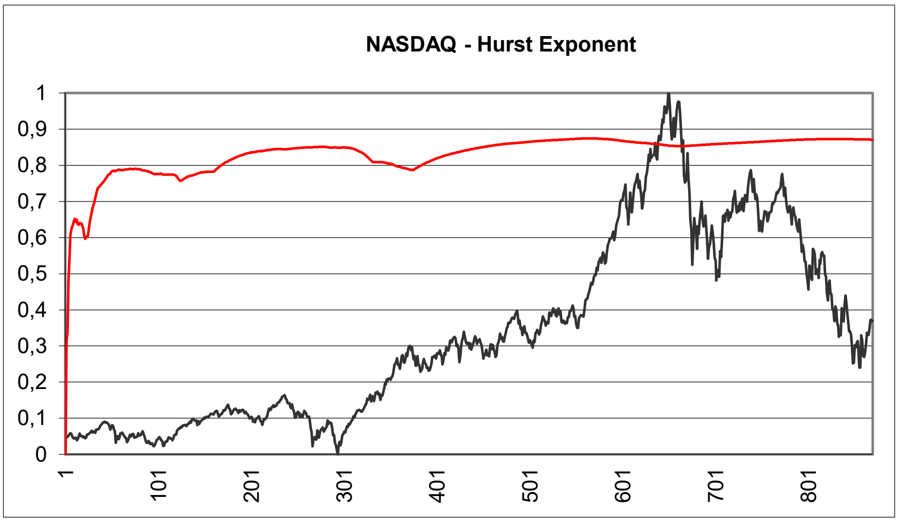

Figure 5: The graph NASDAQ - Hurst Exponent

4. Fractals

The theory of chaos includes fractal geometry. Fractal geometry as described by B. Mandelbrot officially began twenty years ago. The study of the fluctuation of market prices led him to found fractal geometry. The character of fractals is the repetition of a motif: we talk about so-called “self-similarity” and “self-relationship” (for example the structure of a whole tree has self-similarity as a structure of main branches, small branches, twigs, etc.). When we enlarge or reduce any part of a fractal formation, it will be similar to the original one. A fractal is a natural phenomenon or a mathematical set that exhibits a repeating pattern that displays at every scale. If the replication is exactly the same at every scale, it is called a self-similar pattern. Fractals in stock market are mentioned in many books (Peters, 1994).

Application of fractals in economy

This theory is included also in so-called “Elliott waves”. The waves have two phases, the first an “impulse” phase and the second a “correction” phase. The impulse phase consists of five breaks marked 1–5 and a correction phase consists of three breaks marked a–c. Figure 6a presents the situation for an initial increase of a time series and Figure 6b for an initial decrease of a time series. These patterns could be found in time series with various samplings; they are self-similar and have self-relationship. Many time series are published in the mass media from the economy branch and the stock market.

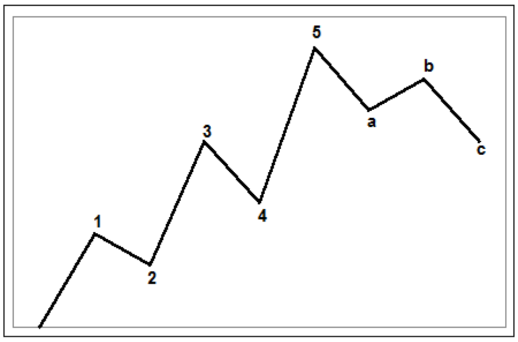

Figure 6a: Elliott wave - increase

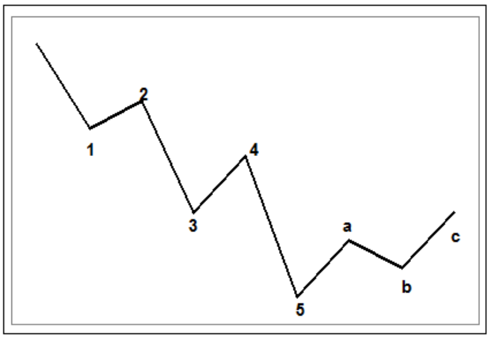

Figure 6b: Elliott wave - decrease

This search for waves is used for evaluation and prediction of time series such as shares, indices, commodities, and currency ratios on stock markets.

The simulation of economy could be done by Tamari chaotic attractor (Tamari, 2015) defined by equations

$$x' = (x - a.y).\cos(z) - b.y.\sin(z)$$

$$y' = (x + c.y).\sin(z) + d.y.\cos(z)$$

$$z' = e + f.z + g.\mathrm{atan}\{ [(1 - u).y] / [(1 - i).x] \}$$ for pricing - spiral version

z' = e + f.z + g.atan{ (1 - u) / (1 - i) x.y }   for wealth - attractor version.

The drawing of Tamari economic attractor and its trajectory is presented in Figure 7 where the dependence of *z* (pricing/wealth) on *x* (output) and *y* (money) is drawn. The exogens are inertia, productivity, printing, adaptation, exchange, indexation, elasticity, expectations, unemployment and interest.

The values of exogens are: a - inertia, b - productivity, c - printing, d - adaptation, e - exchange, f - indexation, g - elasticity/expectations, u - unemployment and I - interest.

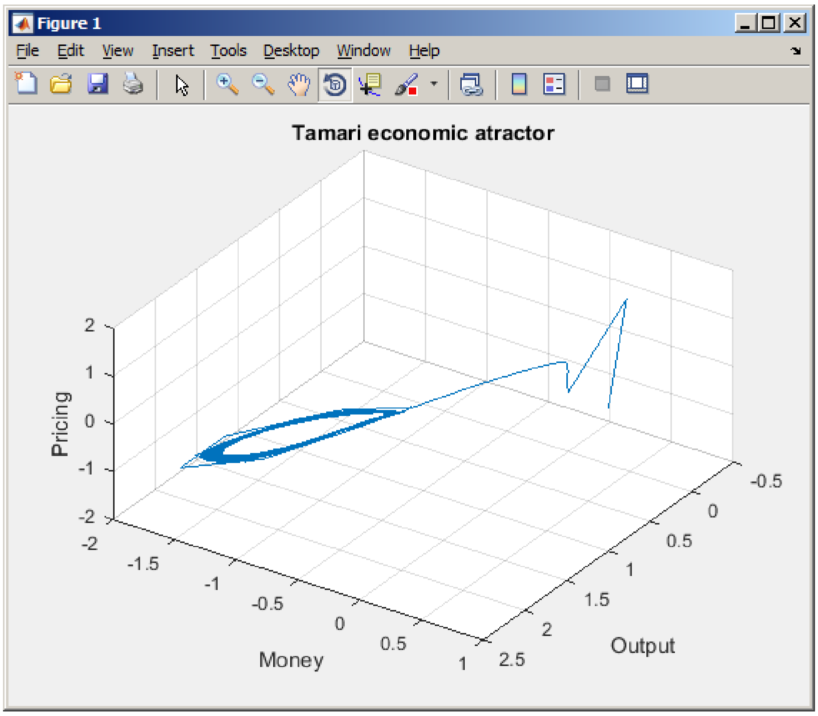

Figure 7: Tamari economic attractor

Tamari attractor enables the study of a country's economy and it is used for simulation and search of country's economy, such as analysis, planning, prediction and comparison with other nations' economies. There are various methods of simulation in economy and on the stock market (Dostal, 2014).

5. Conclusions

The theory of chaos is still under development; nevertheless some outputs are becoming usable in business, such as the calculation of Hurst exponents to evaluate the order or chaos of a business system. The properties of fractals are used for analysis and prediction of time series in stock markets.

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