Chaos and Stock Market

A new approach has been developed to explain the phenomena in financial branch, which the Efficient Market Hypothesis could not deal with. This new Fractal Market Hypothesis is based on the Chaos Theory and it presents an alternative to the Efficient Market Hypothesis. It seams that this new hypothesis better explains the phenomena in financial branch. Hurst and Lyapunov exponent are the part of the Chaos Theory.

The Efficient Market Hypothesis is based on these following assumptions:

- price - all information is reflected by prices
- investor - investors are risk-averse, know what information is important for, and they know how information should be interpreted
- memory - markets do not have any memory, or yesterday's events do not influence today's events
- distribution - returns are independent, normally distributed

The Fractal Market Hypothesis is based on these assumptions:

- price - each individual may interpret information in different ways and at different times
- investor - there are evidences indicating that investors are not rational, they may not know how to interpret all the known information, investors tend to be risk-seeking when losses are involved
- memory - investors are influenced by what has happened, their expectations about the future are shaped by their recent experiences
- distribution - a number of empirical studies on probability distribution of price changes indicate that price changes are not normally distributed (prices fall faster than they rise)

Hurst exponent (H) determines the rate of chaos. If the probability distribution of the system is not normal, or close enough, a nonparametric analysis is needed. Hurst exponent is such a method in the case of time series. Hurst exponent can distinguish fractal from random time series, or find the long memory cycles. If the value of H = 0.5, the time series is normally distributed, or has no long memory process. If ($H_i < H_{i+1}$), the time series is an antipersistent or mean reverting time series. If the time series has been up in the previous period, it is more likely to be going down in the next period, and vice versa. When ($H_i > H_{i+1}$), the time series is persistent or trend reinforcing time series. If the time series has been up in the previous period, it is more likely to be going up in the next period, and vice versa. Hurst exponent can measure how "jagged" a time series is. The lower the H value is, the more jagged the time series is. The higher the H value is, the more apparent the trend is, and the less jagged the time series is. The fractal dimension is calculated as a value = 2.0 – H. The constant c influences the value of H and it was set to 1.0. Hurst algorithm for obtaining this exponent

$$\bar{x}(\tau) = \frac{1}{\tau}\sum_{i=1}^{\tau} x_i ,$$

from a time series is as follows:

$$X(t,\tau) = \sum_{i=1}^{t} (x_i - \bar{x}(\tau)) , \qquad t = 1,2......,\tau;$$

$$S(\tau) = \sqrt{\frac{1}{\tau}\sum_{i=1}^{\tau}(x_i - \bar{x}(\tau))^2} \; ,$$

$$R(\tau) = \max_{1 \le t \le \tau} X(t,\tau) - \min_{1 \le t \le \tau} X(t,\tau) \, ,$$

$$H(\tau) = \log(R(\tau)/(S(\tau) * c)) / \log(\tau) \, .$$

Lyapunov exponent (L) determines the rate of predictability. A positive Lyapunov exponent indicates chaos and it sets the time scale which makes the state of prediction possible. The bigger the largest positive Lyapunov exponent is, the more rapid the loss of predictive "power", and the less the prediction time for the time series is. The predictability is calculated as a value = 1.0 / L. Wolf et al give an algorithm for obtaining the largest Lyapunov exponent from a time series (The calculation of neighbouring orbit D' from a fiducial orbit D is not simple.

$$L(t) = \frac{1}{t}\sum_{i=0}^{t}\log_2\left[\frac{D'(t_{i+1})}{D(t_i)}\right].$$

There are shown some representative results of calculation of Hurst and Lyapunov exponents concerning the part of sinus curve Fig.1., further of Microsoft, Co. title Fig.2. and NASDAQ index Fig.3. of the America stock market. The presented chart for t = t_{max} and graphs confirm theory mentioned above.

$t = t_{max}$	(+) Hurst exponent	Fractal dimension	(-) Lyapunov exponent	Predictability [day]
(x) Sinus	0.554	1.446	0.128	7.83
(x) NASDAQ	0.871	1.129	0.081	12.31
MSFT	0.734	1.266	0.431	2.32

Table 1. Values of exponents

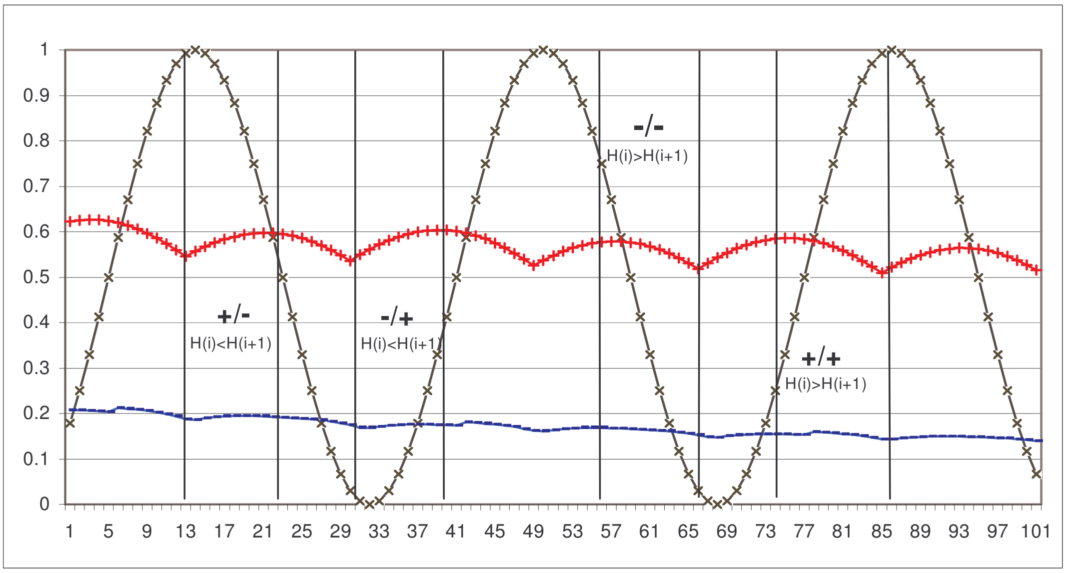

Fig.1. Sinus – Hurst and Lyapunov exponent

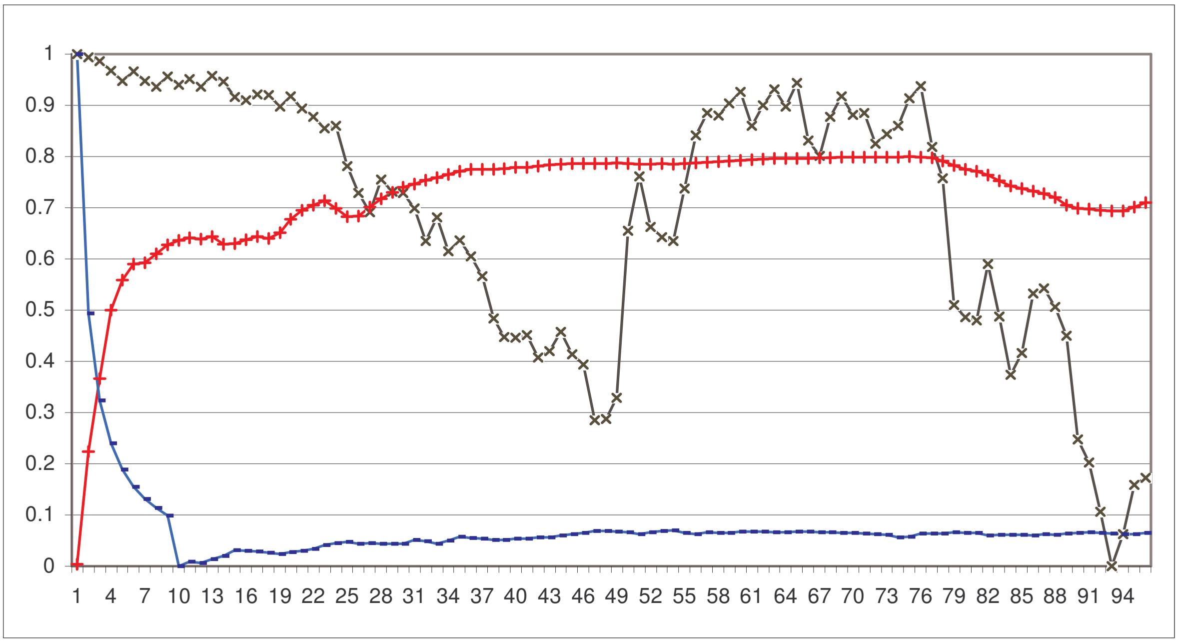

Fig.2. MSFT – Hurst and Lyapunov exponent

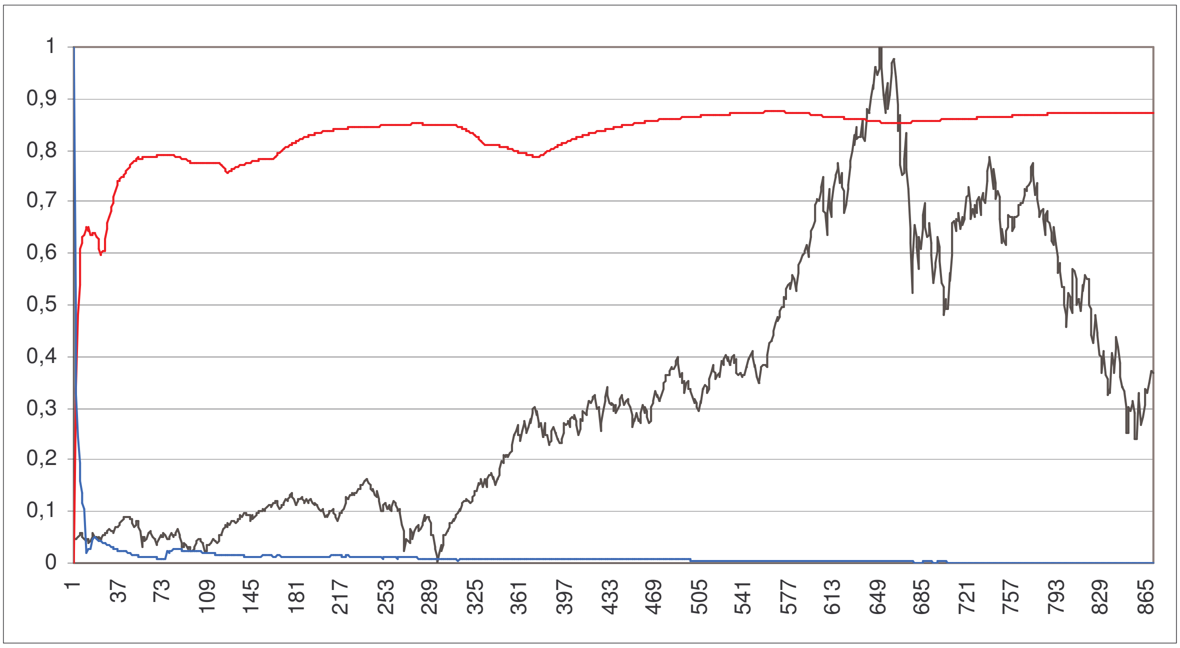

Fig.3. NASDAQ - Hurst and Lyapunov exponent

(The values of MSFT and NASDAQ were normalised so that 1.0 in graph could correspond to maximum value).

The calculation of Hurst exponent or fractal dimension enables us to evaluate how chaotic the time series is. The calculation of Lyapunov exponent or predictability enables us to evaluate the reliability of prediction. If the chaos analyses provide us with good results (the time series has a memory effect and high predictability) it would be advisable to continue with calculations of predictions of time series.