Energy Time Series and Chaos

The theory of chaos and fractal geometry are new theories. It seems that the new theories explain the complicated phenomena which contain non-linearity in a more effective way. These theories can enable us to analyse the time series which are created by complicated social, psychological, economic and financial phenomena. Hurst and Lyapunov exponents are parts of these theories. Hurst exponent enables us to set up the level of chaos and fractal dimension of time series. Lyapunov exponent set up us the "power" of prediction and its reversed value quotes the predictability.

Hurst exponent (H) determines the rate of chaos. If the probability distribution of the system is not normal, or close enough, a nonparametric analysis is needed. Hurst exponent is such a method in the case of time series. Hurst exponent can distinguish fractal from random time series, or find the long memory cycles. If the value of H = 0.5, the time series is normally distributed, or has no long memory process. If ($H_i < H_{i+1}$), the time series is an antipersistent or mean reverting time series. If the time series has been up in the previous period, it is more likely to be going down in the next period, and vice versa. When ($H_i > H_{i+1}$), the time series is persistent or trend reinforcing time series. If the time series has been up in the previous period, it is more likely to be going up in the next period, and vice versa. Hurst exponent can measure how "jagged" a time series is. The lower the H value is, the more jagged the time series is. The higher the H value is, the more apparent the trend is, and the less jagged the time series is. The fractal dimension is calculated as a value = 2.0 – H. The constant c influences the value of H and it was set to 1.0. Hurst algorithm for obtaining this exponent from a time series is as follows:

$$\bar{x}(\tau) = \frac{1}{\tau}\sum_{i=1}^{\tau} x_i \,,$$

$$X(t,\tau) = \sum_{i=1}^{t} (x_i - \bar{x}(\tau)) \,, \qquad t = 1,2......,\tau;$$

$$S(\tau) = \sqrt{\frac{1}{\tau}\sum_{i=1}^{\tau} (x_i - \bar{x}(\tau))^2} \,,$$

$$R(\tau) = \max_{1\le t\le\tau} X(t,\tau) - \min_{1\le t\le\tau} X(t,\tau) \,,$$

$$H(\tau) = \log(R(\tau)/(S(\tau)*c))/\log(\tau) \,.$$

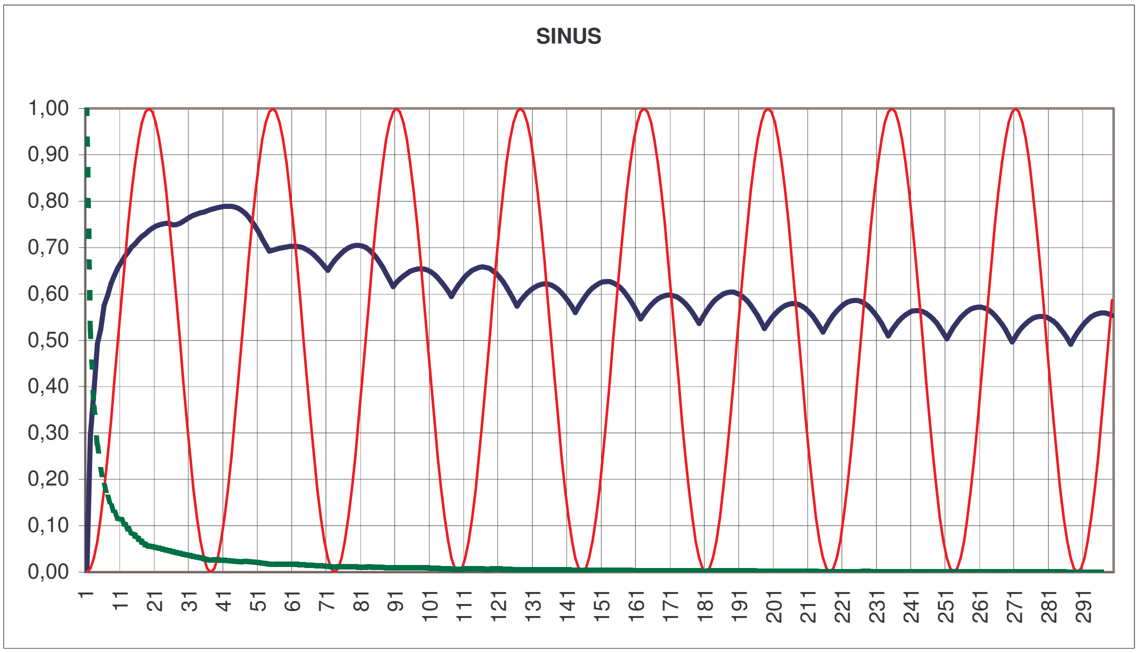
Lyapunov exponent (L) determines the rate of predictability. A positive Lyapunov exponent indicates chaos and it sets the time scale which makes the state of prediction possible. The bigger the largest positive Lyapunov exponent is, the more rapid the loss of predictive "power", and the less the prediction time for the time series is. The predictability is calculated as a value = 1.0 / L. Wolf et al give an algorithm for obtaining the largest Lyapunov exponent from a time series (The calculation of neighbouring orbit D' from a fiducial orbit D is not simple.

$$L(t) = \frac{1}{t}\sum_{i=0}^{t} \log_2 \left[\frac{D'(t_{i+1})}{D(t_i)}\right].$$

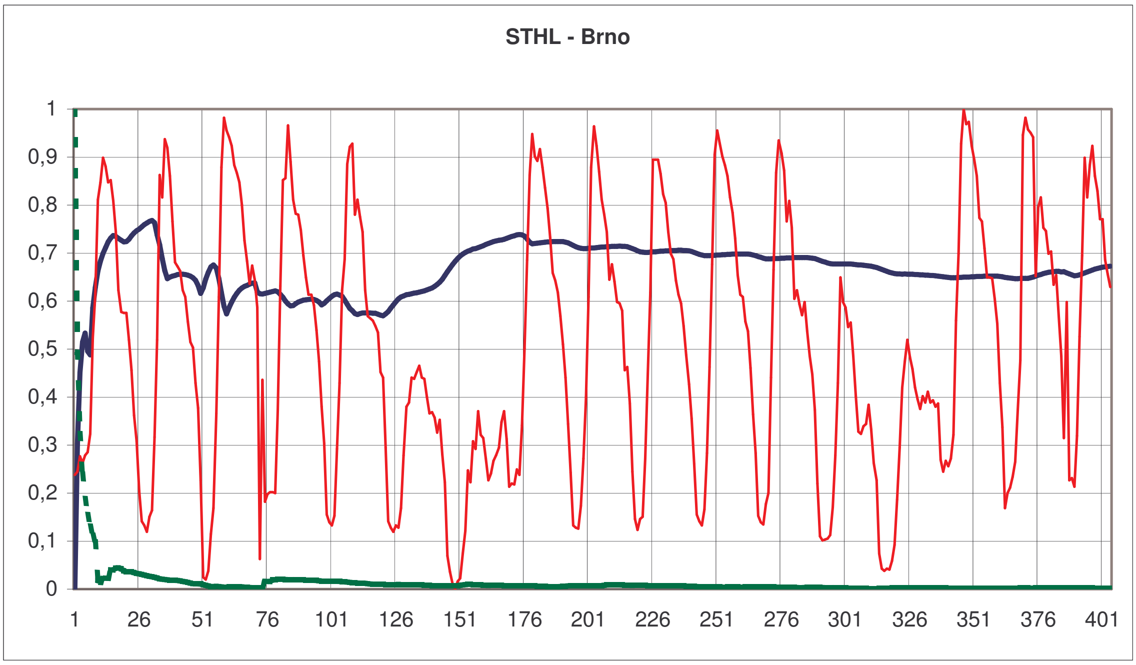
There are shown some representative results of calculation of Hurst (full strong line) and Lyapunov (dashed strong line) exponents concerning the part of sinus curve Fig.1and heat consumption - short term heat load (STHL) in Brno Fig.2. and Olomouc Fig.3 and Fig.4. The tab.1. for t = t_{max} and graphs confirm theory mentioned above.

$t = t_{max}$	(+) Hurst exponent	Fractal dimension	(-) Lyapunov exponent	Predictability [day]
(x) Sinus	0.554	1.446	0.128	7.830
(x) Brno	0.674	1.326	0.259	3.848
(x) Olomouc I.	0.776	1.223	1.003	0.996
(x) Olomouc II.	0.674	1.325	1.071	0.933

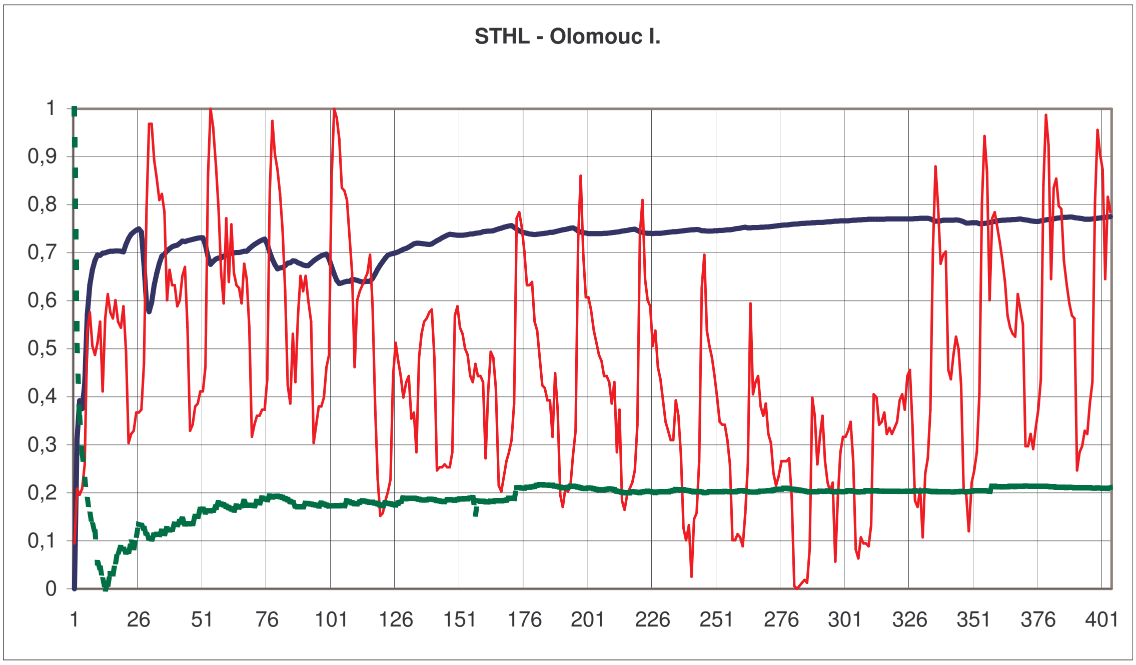
Fig.1. Hurst and Lyapunov exponent values



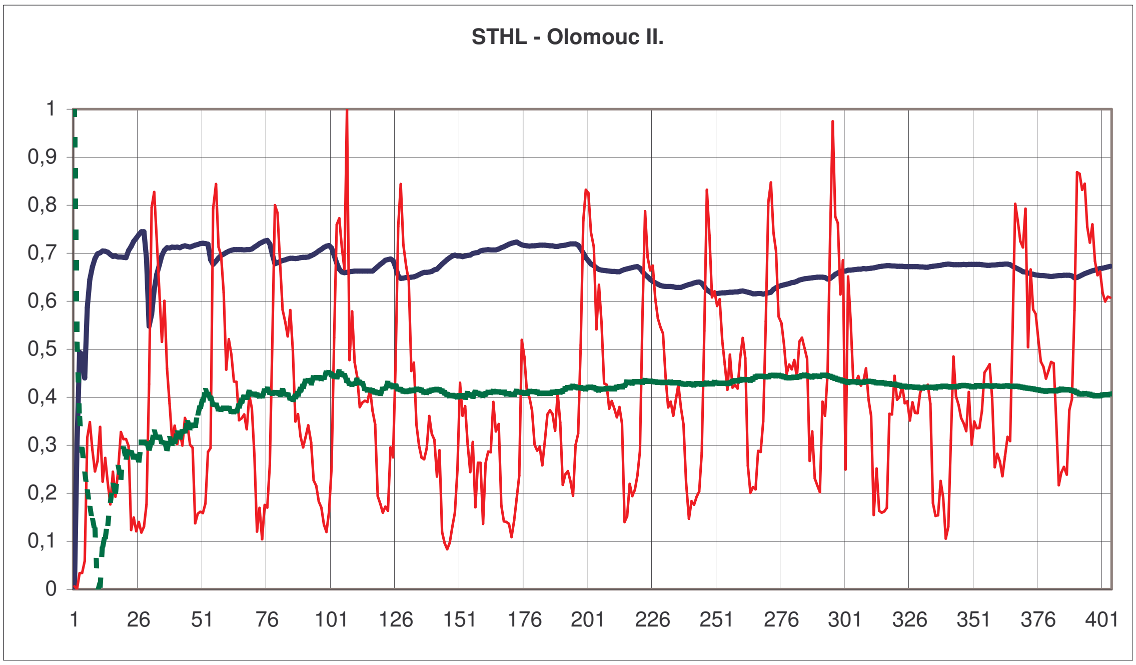
Obr.1. Sinus - Hurst and Lyapunov exponent



Obr.2. STHL - Brno - Hurst a Lyapunov exponent



Obr.3. STHL – Olomouc I. - Hurst a Lyapunov exponent



Obr.4. STHL – Olomouc II. - Hurst a Lyapunov exponent

(The values of STHL were normalised so that 1.0 in graph could correspond to maximum value).
The calculation of Hurst exponent or fractal dimension enables us to evaluate how chaotic the time series is. The calculation of Lyapunov exponent or predictability enables us to evaluate the reliability of prediction. If the chaos analyses provide us with good results (the time series has a memory effect and high predictability) it would be advisable to continue with calculations of predictions of time series.